# BROOKFIELD PUBLIC SCHOOLS 

MATHEMATICS CURRICULUM FRAMEWORK

MARCH 2007

## Mathematics <br> Philosophy/Beliefs

We believe that a strong foundation in mathematics is necessary to solve everyday problems, to think more critically, and to function in a technological world. Learning to imagine, reason intuit, question and prove are higher-order processes that must be developed through a sound program of mathematics.

It is necessary that the mathematics program be meaningful to students. In order to accomplish this, activities must focus on real-world problems, including the applications and strategies needed to solve them.

The process of mathematics, rather than a one-answer and one-method strategy, is very important and must be regularly reinforced. A conceptual focus and understanding of mathematical processes will enable students to be more effective problem solvers and thinkers.

We believe that instruction in mathematics must be varied. The text is only one of many types of materials. Students need to concretely manipulate objects to build the bridge to an abstract understanding, and they must understand how to use technology to solve more complex problems. Classroom instruction is most effective when students have opportunities to interact with others in cooperative groups, explore concepts on an individual basis, work with a peer, engage in classroom discussions, and become involved in project work.

We believe that electronic technologies - calculators and computers - are essential tools for teaching, learning, and doing mathematics. They furnish visual images of mathematical ideas, they facilitate organizing and analyzing data, and they compute efficiently and accurately. They support investigation in every area of mathematics and support students in focusing on decision-making, reflection, reasoning, and problem solving.

Student progress in mathematics must be carefully monitored on a regular basis using a variety of assessment methods. Assessment is most effective when it is a part of the instructional process and when it is employed at the end of a learning cycle. Traditional paper-pencil tests and performance-based assessments complement each other to provide formative and summative information about a student's progress. Students need to demonstrate their understanding of mathematical concepts and their ability to apply them.

We believe that a mathematics program must address a variety of topics and that students must be taught how those topics interrelate. These topics include algebraic reasoning, numerical and proportional reasoning, geometry and measurement, and working with data. Instruction is ongoing, and topics are regularly repeated for reinforcement.

## BROOKFIELD MATHEMATICS CURRICULUM

## Mathematics Program Goals

As a result of education in the Grades K-12 Mathematics program, students will:

- proficiently apply a range of numerical, algebraic, geometric and statistical concepts and skills to formulate, analyze and solve real-world problems; to facilitate inquiry and the exploration of real-world phenomena; and to support continued development and appreciation of mathematics as a discipline;
- communicate numerical, geometric, algebraic and statistical ideas orally and in written form with models, pictures, graphs and mathematical symbols, and written explanations using a variety of calculator displays, spreadsheets, other related computer software, and other relevant technologies;
- use inductive and deductive reasoning to make, defend and evaluate conjectures and arguments, to justify assertions and verity tentative conclusions, and to solve mathematical problems;
- use mathematical skills and concepts to make and justify decisions and predictions, to identify patterns and trends, to pose questions from data and situations, and to formulate and solve problems;
- identify and use connections within mathematics to identify interrelationships and equivalent representations, to construct mathematical models, and to investigate and appreciate mathematical structure;
- select and use appropriate approaches and tools, including various methods of estimations, for solving computational, geometric, algebraic, probability and statistics problems.
- use mathematical skills and concepts with proficiency and confidence, and appreciate the power and utility of mathematics as a discipline and as a tool for solving problems in any real-life situation.


## BROOKFIELD MATHEMATICS CURRICULUM

## Hallmarks of Excellent Practice

Hallmarks of excellent practice are the observable, in-class characteristics and actions that denote a high quality program. The hallmarks are noted in instructional research and represent the best pedagogy to build student independence through the gradual release of responsibility from the teacher to the student:

## Focus of Mathematics Instruction:

- Set high expectations for all students
- Ensure early and equitable opportunities to learn mathematics
- Foster the systematic and appropriate use of technological tools
- Actively involve all students
- Use a variety of problem-solving activities
o Applying strategies
o Open-ended problems
o Patterns and relationships
o Real-world situations
- Use a variety of instructional strategies
o Cooperative learning
o Individual exploration
o Peer instruction
o Project work
o Discussions
o Adjusting instruction to accommodate students with diverse needs, abilities, and interests
- Focus on concepts and process, as well as the acquisition of basic and advanced skills
- Use a variety of learning tools
o Manipulatives
o Paper/pencil
o Technology
- Ensure periodic instruction and review of skills
- Use a variety of assessment strategies
o Writing and explaining
o Performance-based
o Traditional paper/pencil
- Integrate and instruct in multiple topics
o Number sense
o Operations
o Estimation and approximation
o Ratios, percents, proportions
o Measurements
o Geometry
o Probability and statistics
o Patterns
o Algebra and functions
o Discrete math - analyze and model a variety of real-world situations that involve recurring relationships, sequences, networks, combinations and permutations

1. Teacher modeling the expected performance through:

- Explicit instruction identifying effective strategies by working with exemplars, high quality examples, in various texts including those available through the use of technology (textbooks, primary sources, student work, and other literature)
- Think-alouds
- Meta-Cognition
- Demonstrations by teachers of their own work as artists, writers, scientists, musicians, craftspeople, readers, mathematicians, hobbyists, poets, researchers, etc.
-Mentors are used as guides to develop student competencies (the works of artists, writers, scientists, mathematicians, poets, researchers, etc.)
- Rubrics define expectations.


## 2. Guided Practice through:

- Activities that build on one another with increasing levels of complexity and decreasing levels of support
- Multiple structures that meet the individual needs of students (differentiating instruction through a mix of whole class, small group and individual activities)
- Strategies or criteria that are adjusted to accommodate differences in students' learning needs
- Teacher as coach/facilitator, providing feedback during activities
- Active learning - exploring/learning by doing and discussions with 'peers
- Conferencing engage students to reflect on their learning processes
- Use of visual tools to track thinking by coding text, the use of post-its, and graphic organizers
- Student use rubrics to guide their work


## 3. Independent Practice through:

- Activities that prompt students to apply new learning
- Self- assessment tools such as rubrics, checklists, and reflective responses
- Routines that cause students to analyze their own work and reflect on their growth as learners.

4. Application opportunities through:

- Activities or performance expectations that require students to apply skills and strategies, and new concepts in real world, authentic performance tasks. This may include the use of technology.


## 5. Assessment practice focuses on:

- The continual assessment of student learning to guide instruction:
- Diagnostic assessment before instruction to guide the instructional learning process
- Formative assessment during instruction to provide specific and frequent feedback to students and teachers and to make adjustments to refocus attention
- Summative assessments at the end of instruction to determine the level of attainment of the learning goals.


## BROOKFIELD MATHEMATICS CURRICULUM

## Hallmarks Continued Essential Understandings

1. Algebraic Reasoning: Patterns and Functions - Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.
2. Numerical and Proportional Reasoning - Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.
3. Geometry and Measurement - Shapes and structures can be analyzed, visualized, and measured and transformed using a variety of strategies, tools and technologies.
4. Working with Data: Probability and Statistics - Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies.

## Mathematics Essential Questions

How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?

How are quantitative relationships represented by numbers?
How do geometric relationships and measurements help us to solve problems and make sense of our world?

How can collecting, organizing, and displaying data help us analyze information and make reasonable and informed decisions?

## Mathematics Content Standards

## Students should:

1.1 Understand and describe patterns and functional relationships.
1.2 Represent and analyze quantitative relationships in a variety of ways.
1.3 Use operations, properties and algebraic symbols to determine equivalence and solve problems.
2.1 Understand that a variety of numerical representations can be used to describe quantitative relationships.
2.2 Use numbers and their properti4es to compute flexibly and fluently, and to reasonably estimate measures and quantities.
3.1 Use properties and characteristics of two - and three - dimensional shapes and geometric theorems to describe relationships, communicate ideas and solve problems.

## BROOKFIELD MATHEMATICS CURRICULUM

3.2 Use spatial reasoning, location and geometric relationships to solve problems.
3.3 Develop and apply units, systems, formulas and appropriate tools to estimate and measure.
4.1 Collect, organize and display data using appropriate statistical and graphical methods.
4.2 Analyze data sets to form hypotheses and make predictions.
4.3 Understand and apply basic concepts of probability.

| $\begin{array}{c}\text { Prekindergarten } \\ \text { ALGEBRAIC REASONING: PATTERNS AND FUNCTIONS }\end{array}$ |  |  |
| :---: | :---: | :--- |
| Patterns and functional relationships can be represented and analyzed using |  |  |
| a variety of strategies, tools and technologies. |  |  |$]$


| Prekindergarten <br> NUMERICAL AND PROPORTIONAL REASONING: Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies. |  |  |
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| How are quantitative relationships represented by numbers? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 2.1 Understand that a variety of numerical representations can be used to describe | a. Use numbers to count, order and compare. | (N1) Count and identify the number of objects in a set. <br> (N2) Compare sets and identify those with more, less and the same amounts. <br> (N3) Describe the position of objects using the terms first and last. |
| quantitative relationships. | b. Share equal parts of a whole object. | (N4) Explore a whole and half of an object. |
| 2.2 Use numbers and their properties to compute flexibly and fluently and to reasonably estimate measures and quantities. | a. Count, adding one or more to the previous amount. | (N5) Count by rote, 1 to 10 <br> (N6) Count as one more object is added to a set of objects. |


| Prekindergarten <br> GEOMETRY AND MEASUREMENT <br> Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How do geometric relationships and measurements help us to solve problems and make sense of our world? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 3.1 Use properties and characteristics of two- and threedimensional shapes and geometric theorems to describe relationships, communicate ideas and solve problems. | a. Identify and sort shapes and solids by physical characteristics. | (G1) Identify and sort simple shapes such as square, rectangle, triangle and circle, and solids such as cube, cylinder, sphere and prism. <br> (G2) Use a variety of materials to construct various shapes and describe their physical attributes. |
| 3.2 Use spatial reasoning, location and geometric relationships to solve problems. | a. Use positional language to describe location, direction and position of objects. | (G3) Use positional language - e.g., under, over, inside, next, near, in front - to describe position and order. <br> (G4) Complete simple shape and jigsaw puzzles. |
| 3.3 Develop and apply units, systems, formulas | a. Sequence events during a limited time period. | (G5) Describe time periods or a sequence of events using terms such as morning, afternoon and night or yesterday, today and tomorrow. |
| and appropriate tools to estimate and measure. | b. Use nonstandard units to estimate measures of length, area and capacity. | (G6) Use nonstandard units and body referents to compare and estimate length, area and capacity. <br> (G7) Sort, estimate and order objects by length or area using comparative language such as more, longer, shorter, taller or bigger to describe relationships. |


| Prekindergarten <br> WORKING WITH DATA: PROBABILITY AND STATISTICS <br> Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies. |  |  |
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| How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 4.1 Collect, organize and display data using appropriate statistical and graphical methods. | a. Make comparisons from information displayed in real graphs. | (D1) Collect, describe, organize, sort and display objects and pictures in real graphs. |
| 4.3 Understand and apply basic concepts of probability. | a. Determine when events are likely to happen again. | (D2) Use patterns to describe some events that repeat. <br> (D3) Identify events related to personal experiences as likely or unlikely to happen. |


| Kindergarten <br> ALGEBRAIC REASONING: PATTERNS AND FUNCTIONS <br> Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies. |  |  |
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| How do patterns and functions help us describe data and physical phenomena and solve a variety of problems? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 1.1 Understand and describe patterns and | a. Sort and classify objects using attributes. | (A1) Sort and classify objects by size, shape, color, texture, use, position and orientation and describe the reason for the action. |
| functional relationships. | b. Identify a pattern and describe the rule using the physical attributes or position of objects in a sequence. | (A2) Recognize, copy and extend patterns of sounds, colors, shapes, textures and numbers in a variety of contexts and describe the rule of the pattern. <br> (A3) Make comparisons and describe qualitative and quantitative changes of a given pattern (more, less, bigger, smaller, longer, one more, one less). |


| Kindergarten <br> NUMERICAL AND PROPORTIONAL REASONING: Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies. |  |  |
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| How are quantitative relationships represented by numbers? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 2.1 Understand that a variety of numerical representations can be used to describe quantitative relationships. | a. Use numbers to count, order, compare, label, locate and measure. | (N1) Use numbers to locate, order, label and measure. <br> (N2) Identify the numerals 1-10 and match sets of objects to the numerals. <br> (N3) Compare sets using the terms "more," "less" or "the same" and order sets from least to greatest. <br> (N4) Identify ordinal position of objects, first through fifth, and last. <br> (N5) Act out story problems and solve practical problems using objects. |
|  | b. Share equal parts of an object. | (N6) Use a variety of models to identify a whole and a half of an object. <br> (N7) Compare two parts of a whole and describe the parts as closer to a whole, or closer to very little. <br> (N8) Recognize that two halves can be put together to make a whole. |
|  | c. Share a set of objects that is divided into groups with equal amounts. | (N9) Share a set of objects considered to be a whole by forming two smaller sets that have equal amounts. |
| 2.2 Use <br> numbers and their properties to compute flexibly and fluency and to reasonably estimate measures and quantities. | a. Count, adding one more to the previous number, and group and count by ones and tens. | (N10) Count to and past 10 to 20, then to 30, and group and count objects by 10. <br> (N11) Estimate the amount of objects in a set using 10 as a benchmark, and then count to determine if the amount is more or less than 10. <br> (N12) Identify sets and numbers, which are equal and one more. <br> (N13) Recognize and name pennies and dimes; count and trade pennies for objects. |


| Kindergarten <br> GEOMETRY AND MEASUREMENT <br> Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies. |  |  |
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| How do geometric relationships and measurements help us to solve problems and make sense of our world? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 3.1 Use properties and characteristics of two- and threedimensional shapes and geometric theorems to describe relationships, communicate ideas and solve problems. | a. Identify and sort shapes and solids by physical characteristics. | (G1) Sort, order, compare and use comparative language to describe small sets of objects sequenced by size, length, area and volume. <br> (G2) Identify, sort and compare two- and threedimensional shapes and solids in the environment, such as triangles, squares, rectangles, circles, cubes, spheres, cylinders and cones. <br> (G3) Use a variety of materials to create geometric shapes and solids and build copies of simple shapes and designs by direct observation and by visual memory. |
| 3.2 Use spatial reasoning, location and geometric relationships to solve problems. | a. Use positional language to describe location, direction and position of objects. | (G4) Describe the position, location and direction of objects, or parts of objects, using terms such as inside, outside, top, bottom, close, closer, etc. |
| 3.3 Develop and apply units, systems, | a. Use calendars and clocks to measure and record time. | (G5) Locate a date on the calendar (yesterday, today and tomorrow) and sequence events using terms like before and after. |
| formulas and appropriate tools to estimate and measure. | b. Use nonstandard units to estimate measures of length, area, temperature, weight and capacity. | (G6) Estimate the number of objects in a handful, and then count to verify. <br> (G7) Estimate the amount of objects in a set using benchmarks of 10 , and count to determine if the estimate is more or less. <br> (G8) Explore, describe and discuss strategies to estimate length, area, temperature and weight using nonstandard units to compare. <br> (G9) Explore using everyday objects as nonstandard units to measure length, area and capacity. <br> (G10) Compare the weight of two objects using a balance scale and identify which is heavier. |


| Kindergarten <br> WORKING WITH DATA: PROBABILITY AND STATISTICS <br> Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies. |  |  |
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| How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 4.1 Collect, organize and display data using appropriate statistical and graphical methods. | a. Visualize information and make comparisons about information displayed in real and picture graphs. | (D1) Pose questions about personal information, experiences and environment. <br> (D2) Explore ways to record and organize data using tallies and tables. <br> (D3) Construct real graphs and picture graphs and describe the data using the terms more, less and same. <br> (D4) Organize information through systematic counting, sorting, making lists and graphic organizers. |
| 4.2 Analyze data sets to form hypotheses and make predictions. | a. Extend different types of patterns and make predictions. | (D5) Identify visual, auditory and physical patterns and extend to make predictions. |
| 4.3 Understand and apply basic concepts of probability. | a. Observe the frequency of real-world events and identify the likelihood of future events. | (D6) Describe the likelihood of events related to personal experiences. <br> (D7) Engage in simple probability activities and discuss the results. |


| Grade 1 <br> ALGEBRAIC REASONING: PATTERNS AND FUNCTIONS <br> Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How do patterns and functions help us describe data and physical phenomena and solve a variety of problems? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 1.1 Understand and describe patterns and functional relationships. | a. Examine attributes of objects and describe their relationships. | (A1) Sort, classify and order objects and numbers based on one and two attributes and describe the rule used. <br> (A2) Recognize, extend, describe and create a variety of patterns, and translate the same pattern from one representation (such as color) to another representation (such as shape). <br> (A3) Describe counting patterns and number patterns. <br> (A4) Develop and test generalizations based on observations of patterns and relationships. |
| 1.2 Represent and analyze quantitative relationships in a variety of ways. | a. Represent the result of counting, combining and separating sets of objects using number sentences. | (A5) Model real-life situations that involve addition and subtraction of whole numbers using objects, pictures and open sentences. |
| 1.3 Use operations, properties and algebraic symbols to determine equivalence and solve problems. | a. Identify quantities as equivalent or nonequivalent. | (A6) Demonstrate balance or equivalence using models. |


| Grade 1 <br> NUMERICAL AND PROPORTIONAL REASONING: Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies. |  |  |
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| How are quantitative relationships represented by numbers? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 2.1 Understand that a variety of numerical representations can be used to describe quantitative relationships. | a. Represent and order two-digit numbers as groups of tens and ones in the base ten place value system. | (N1) Estimate and describe quantity with benchmark amounts such as 0,10 and 100. <br> (N2) Represent two-digit numbers on number lines and using models. <br> (N3) Determine and compare values and trade with sets of pennies and dimes. <br> (N4) Identify ordinal position of objects, first through tenth. |
|  | b. Identify and compare equal parts of a whole. | (N5) Identify and represent $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ of a whole and identify portions that are not halves, thirds or fourths. (N6 Compare parts of a whole object and estimate whether they are closer to a very little, one half or one whole. <br> (N7) Make a whole of equal-sized parts of familiar objects. |
|  | c. Partition a set of objects into smaller groups with equal amounts. | (N8) Identify half of a small set of objects considered to be the whole. |
|  | d. Describe relationships between quantities using ratios. | (N9) Describe patterns with simple ratios using familiar contexts, such as 1 cat has 4 legs, 2 cats have 8 legs. |
| 2.2 Use <br> numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities | a. Count by groups, add one more to the grouping and compare values of groups. | (N10) Count whole numbers to 100. <br> (N11) Identify, read and write numerals to 100 and beyond. <br> (N12) Group and skip count by $2 \mathrm{~s}, 5 \mathrm{~s}$ and 10 s . <br> (N13) Count on from a given amount, orally and with models, and count back from 10. <br> (N14) Identify 1 more and 1 less and explore 10 more and 10 less than a number. |
|  | b. Add by counting and combining and subtract by separating, comparing or counting on. | (N15) Write number sentences and use objects and pictures to model and solve addition and subtraction story problems. <br> (N16) Develop, describe and use a variety of strategies to add and subtract one-digit numbers. (N17) Explore finding the sum of two two-digit numbers using models and counting strategies. <br> (N18) Identify reasonable answers to problems that reflect real-world experiences. |


| Grade 1 <br> GEOMETRY AND MEASUREMENT <br> Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How do geometric relationships and measurements help us to solve problems and make sense of our world? |  |  |
| Students <br> Should... | Performance Standards | Expected Performances |
| 3.1 Use <br> properties and characteristics of two- and threedimensional shapes and geometric theorems to describe relationships, communicate ideas and solve problems. | a. Classify shapes and solids by common characteristics. | (G1) Sort, build, name and draw two- and threedimensional objects. <br> (G2) Use a variety of materials to create two- and three-dimensional designs and copy them from visual memory. <br> (G3) Create and explore shapes and designs with a line of symmetry. |
| 3.2 Use spatial reasoning, location and geometric relationships to solve problems. | a. Describe, name and interpret direction and position of objects. | (G4) Indicate relative position, direction and location with terms such as inside, outside, top, bottom, left and right. |
| 3.3 Develop and apply units, systems, formulas and appropriate tools to estimate and measure. | a. Plan and sequence events. | (G5) Estimate and compare the length of time needed to complete tasks using terms such as longer or shorter. <br> (G6) Use the calendar to identify dates, days, weeks and months and to plan and sequence events. <br> (G7) Tell time to the hour with analog and digital clocks. |
|  | b. Estimate length, area, volume, weight and temperature using nonstandard units. | (G8) Use physical referents to make estimates and to determine and describe the reasonableness of answers to measurement problems. <br> (G9) Use estimation, physical referents and nonstandard units to sort and compare objects. |
|  | c. Use standard units of measure to communicate measurement in a universal manner. | (G10) Explore using the standard units of inch and centimeter to estimate and measure length. |


| Grade 1 <br> WORKING WITH DATA: PROBABILITY AND STATISTICS <br> Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 4.1 Collect, organize and display data using appropriate statistical and graphical methods. | a. Collect, organize, record and describe data. | (D1) Pose questions and collect, organize, record and describe data using tallies, tables, real graphs, picture graphs, glyphs (coded pictures) and bar graphs. |
| 4.2 Analyze data sets to form hypotheses and make predictions. | a. Organize data in tables and graphs and make comparisons of the data. | (D2) Use various methods to organize information including lists, systematic counting, sorting, graphic organizers and tables. <br> (D3) Use comparative language to describe the data in tables and graphs. |
| 4.3 Understand and apply basic concepts of probability. | a. Determine the likelihood of certain events through simple experiments and observations of games. | (D4) Observe, record, graph and describe the results of simple probability activities and games. <br> (D5) Describe and explain the likelihood of various events in the students' world. |


| Grade 2 <br> ALGEBRAIC REASONING: PATTERNS AND FUNCTIONS <br> Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How do patterns and functions help us describe data and physical phenomena and solve a variety of problems? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 1.1 Understand and describe patterns and functional relationships. | a. Describe and extend patterns. | (A1) Describe and classify data and objects based on more than one attribute. <br> (A2) Use patterns and the rules that describe them to identify a missing object, objects with common or different attributes, and the complement of a set of objects. <br> (A3) Explore a variety of ways to describe and write rules for patterns. |
|  | b. Analyze change in terms of quantity and quality using patterns. | (A4) Explore and describe number patterns including odd and even numbers, counting by $2 \mathrm{~s}, 5 \mathrm{~s}, 10 \mathrm{~s}, 100 \mathrm{~s}$ and counting on by 10 . (A5) Make comparisons of data and analyze observable changes using qualitative and quantitative descriptions. |
| 1.2 Represent and analyze quantitative relationships in a variety of ways. | a. Represent real-life situations using number sentences. | (A6) Model situations involving addition and subtraction of whole numbers using objects, pictures, symbols and open sentences. |
| 1.3 Use operations, properties and algebraic symbols to determine equivalence and solve problems. | a. Represent quantities that have the same value with an equal sign. | (A7) Demonstrate understanding of the $=$ sign as an equality symbol. |


| Grade 2 <br> NUMERICAL AND PROPORTIONAL REASONING: Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies. |  |  |
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|  |  |  |
| How are quantitative relationships represented by numbers? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 2.1 Understand that a variety of numerical representations can be used to describe quantitative relationships. | a. Represent threedigit numbers as groups of hundreds, tens and ones in the base ten place value system. | (N1) Use place value models and pictures to represent two- and three-digit numbers and write numbers in expanded and regrouped forms. <br> (N2) Locate, label and order two- and three-digit numbers using place value models, pictures and number lines. <br> (N3) Use place value models, pictures and number lines to identify 10 more and 10 less and 100 more and 100 less than a number. <br> (N4) Count with and trade pennies, dimes and dollars and determine and compare values. |
|  | b. Represent fractions by sharing portions of equal size as parts of a whole or parts of a set. | (N5) Model and describe equal parts of a whole as unit fractions $\frac{1}{2}$ through $\frac{1}{10}$. <br> (N6) Use models and familiar objects to estimate, compare and order unit fractions ( $\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)$ of a whole. (N7) Estimate and use counting and grouping of objects to find equal parts of a small set of counting objects, such as $\frac{1}{2}, \frac{1}{3}$ or $\frac{1}{4}$ of 12 cookies. <br> (N8) Explore equivalent fractions using models. |
|  | c. Recognize that the denominator of a fraction tells how many equal parts an object or a set has been divided into, and that the numerator indicates how many of the parts are being considered. | (N9) Identify and build models of fractional parts of a whole (such as $\frac{3}{4}$ ), other than unit fractions. <br> (N10) Explore and describe addition with like denominators and write matching fraction sentences using models. |
|  | d. Describe relationships between quantities using ratios. | (N11) Describe simple ratios in patterns using models and pictures (e.g., in a pattern of green, green, red blocks, there are always two green blocks for each red block). |


| 2.2 Use <br> numbers and <br> their properties <br> to compute <br> flexibly and <br> fluently, and to <br> reasonably <br> estimate <br> measures and <br> quantities. | a. Develop fact <br> families of basic <br> facts using the <br> inverse relationship <br> of addition and <br> subtraction. | (N12) Recall basic addition and subtraction facts. <br> (N13) Identify reasonable answers and solve addition <br> and subtraction problems involving real-world <br> experiences. |
| :--- | :--- | :--- |
|  |  | b. Explore the <br> relationship of <br> multiplication and <br> division through a <br> variety of methods. |
|  | (N14) Explore multiplication by extending number <br> patterns, skip counting, combining repeated addends, <br> building models of groups the same size and using <br> arrays and pictures. <br> (N15) Explore the connection between multiplication <br> and division using models and pictures of groups and <br> arrays. |  |
|  |  | c. Identify and use <br> equivalent <br> representations of <br> numbers to estimate <br> and compute. |
| (N16) Compare and round numbers to the nearest 10 <br> using place value models and number lines. <br> (N17) Explore and describe strategies for <br> representing, estimating, adding and subtracting two <br> two-digit numbers with and without regrouping. <br> (N18) Recognize when an estimate is appropriate and <br> use estimation strategies that result in identifying a <br> reasonable answer to a problem. |  |  |


| Grade 2 <br> GEOMETRY AND MEASUREMENT <br> Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How do geometric relationships and measurements help us to solve problems and make sense of our world? |  |  |
| Students <br> Should... | Performance Standards | Expected Performances |
| 3.1 Use <br> properties and characteristics of two- and threedimensional shapes and geometric theorems to describe relationships, communicate ideas and solve problems. | a. Identify shapes as the same when there are changes in position. | (G1) Explore translations (slides), reflections (flips) and rotations (turns) of simple polygons using manipulative materials. <br> (G2) Build and identify shapes that have one or more lines of reflective symmetry or that can be divided into two congruent parts. <br> (G3) Explore filling a two-dimensional region with different shapes. |
| 3.2 Use spatial reasoning, location and geometric relationships to solve problems. | a. Recognize and use geometric relationships to solve problems. | (G4) Explore combining and subdividing polygons and solids with manipulative materials and reconstruct them from visual memory. (G5) Build, describe, draw and identify polygons, solids and other two- and threedimensional objects found in the environment. |
| 3.3 Develop and apply units, systems, formulas and | a. Estimate and measure the length of time to complete activities and tasks. | (G6) Use the calendar to write and solve problems involving time. <br> (G7) Tell time to the half-hour, and explore time to the quarter-hour (analog and digital). |
| appropriate tools to estimate and measure. | b. Measure through direct comparison and through repetition of units. | (G8) Develop and use nonstandard referents and standard benchmarks to estimate and measure length, area, weight, capacity and volume. (G9) Identify reasonable estimates and describe the strategies used to determine the estimates. (G10) Explore using measurement tools such as thermometers, basic rulers and balance scales to measure temperature, length and weight. |


| WORKING WITH DATA: <br> DRade 2 <br> Data can be analyzed to make informed decisions using <br> a variety of strategies, tools and technologies. |  |  |  |
| :--- | :--- | :--- | :---: |
| How can collecting, organizing and displaying data help us analyze information and <br> make reasonable predictions and informed decisions? |  |  |  |
| Students <br> should... | Performance Standards | Expected Performances |  |
| 4.1 Collect, <br> organize and <br> display data <br> using <br> appropriate <br> statistical and <br> graphical <br> methods. | a. Construct graphs from <br> data, then make <br> comparisons and draw <br> conclusions. | (D1) Pose questions and systematically <br> collect, sort, organize, record and analyze <br> data using tables, charts and picture and <br> bar graphs. <br> (D2) Use comparative terms to describe <br> data. |  |
| 4.2 Analyze data <br> sets to form <br> hypotheses and <br> make <br> predictions. | a. Determine patterns and <br> make predictions from data <br> displayed in tables and <br> graphs. | (D3) Investigate combinations using <br> models. |  |
| 4.3 Understand <br> and apply basic <br> concepts of <br> probability. | a. Analyze data gathered <br> from experiments and <br> identify the likelihood of <br> future events. | (D4) Discuss the likelihood of various <br> events, state possibilities, make <br> predictions and test them in practical <br> situations. <br> (D5) Conduct probability experiments and <br> record the results in tables and graphs. |  |


| Grade 3 <br> Patterns and functional relationships can be represented and analyzed using <br> a variety of strategies, tools and technologies. |  |
| :--- | :--- | :--- |
| How do patterns and functions help us describe data and physical |  |
| phenomena and solve a variety of problems? |  |


| Grade 3 <br> NUMERICAL <br> can be expressed numerically in multiple ways in order to make connections and <br> simplify calculations using a variety of strategies, tools and technologies. |  |  |
| :--- | :--- | :--- |
| How are quantitative relationships represented by numbers? |  |  |


|  | b. Approximate solutions to problems involving computation through the use of efficient methods. | (N12) Estimate, add and subtract with twoand three- digit numbers using a variety of strategies. <br> (N13) Use estimation strategies to determine and justify the reasonableness of a computational answer. <br> (N14) Recognize when an estimate is appropriate and determine whether an estimation strategy will result in an over-or underestimate. |
| :---: | :---: | :---: |
|  | c. Solve multiplication and division problems using rectangular arrays, number patterns, skip counting and repeated addends. | (N15) State the multiplication and division facts with factors of $1,2,3,4,5$ and 10. <br> (N16 Explore division problems with and without remainders. <br> (N17) Write and solve multiplication and division story problems and match to number sentences (equations). <br> (N18) Use models and pictures of sets and arrays to represent multiplication and division of two- and three-digit numbers by one-digit numbers. |
|  | d. Compare fractions, identify equivalent fractions and add and subtract fractions with like and unlike denominators using models and pictures. | (N19) Construct and use models to identify equivalent fractions and to compare and order fractions with like and unlike denominators of $2,3,4,5,6$ and 8 . <br> (N20) Identify patterns with equivalent ratios such as 3 out of 6 crayons are red or 4 out of 8 crayons are red are the same as 1 out of 2 crayons is red. <br> (N21) Construct and use models to add and subtract fractions with like and unlike denominators and write fraction sentences to match the models. <br> (N22) Write and solve story problems that involve fractions. |


| Grade 3 <br> GEOMETRY AND MEASUREMENT <br> Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How do geometric relationships and measurements help us to solve problems and make sense of our world? |  |  |
| Students Should... | Performance Standards | Expected Performances |
| 3.1 Use properties and characteristics of two- and threedimensional shapes and geometric theorems to describe relationships, communicate ideas and solve problems. | a. Classify and compare polygons and solids using various attributes. | (G1) Sort polygons and solids through using characteristics such as the relationship of sides (parallel, perpendicular), kinds of angles (acute, right and obtuse), symmetry and congruence. <br> (G2) Describe similarities and differences of two- and three-dimensional shapes in the environment using physical features such as number of sides, number of angles, lengths of sides and straight and curved parts. <br> (G3) Investigate ways to tile or tessellate a region or shape using various polygons. |
| 3.2 Use spatial reasoning, location and geometric relationships to solve problems. | a. Represent location on simple maps. | (G4) Draw and interpret simple maps using coordinate systems and shapes or pictures. |
| 3.3 Develop and apply units, systems, formulas and appropriate | a. Plan events and make schedules. | (G5) Tell time to the minute, using analog and digital clocks, and identify AM and PM. (G6) Use calendars and clocks to plan and sequence events. |
| tools to estimate and measure. | b. Determine and use different tools and units appropriate for specific measurement tasks. | (G7) Develop and explain strategies for using nonstandard and standard referents to estimate measurements of length, area, weight, temperature, volume and capacity. (G8) Explore strategies for estimating and measuring the perimeters, areas and volumes of irregular shapes and solids. <br> (G9) Describe and use estimation strategies that can identify a reasonable answer to a measurement problem when an estimate is appropriate. |

## Grade 3

WORKING WITH DATA: PROBABILITY AND STATISTICS
Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies.

| How can collecting, organizing and displaying data help us analyze information and <br> make reasonable predictions and informed decisions? |  |  |
| :--- | :--- | :--- |
| Students <br> should... | Performance <br> Standards | Expected Performances |
| 4.1 Collect, <br> organize and <br> display data <br> using <br> appropriate <br> statistical and <br> graphical <br> methods. | a. Design surveys for <br> the collection of data <br> and justify conclusions <br> drawn from the data. | (D1) Pose questions and use a variety of <br> ways to collect, organize and analyze data <br> from samples and surveys. <br> (D2) Display, read, interpret and draw <br> conclusions from data that is represented in <br> a variety of ways including tables, charts, <br> lists, diagrams, line plots and bar graphs. |
| 4.2 Analyze <br> data sets to <br> form <br> hypotheses and <br> make <br> predictions. | a. Analyze data to <br> identify a typical <br> element or event. | (D3) Describe trends in data using range and <br> mode. |
| 4.3 Understand <br> and apply <br> basic concepts <br> of probability. | a. Use samples and <br> simulations to <br> determine probability <br> and to make and test <br> predictions. | (D4) Make predictions and test them by <br> conducting probability experiments and <br> recording results. <br> (D5) Explore the fairness of games <br> involving a variety of spinners and dice. |


| Grade 4 <br> ALGEBRAIC REASONING: PATTERNS AND FUNCTIONS <br> Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How do patterns and functions help us describe data and physical phenomena and solve a variety of problems? |  |  |
| Students Should... | Performance Standards | Expected Performances |
| 1.1 Understand and describe patterns and functional relationships. | a. Classify patterns as repeating or growing. | (A1) Recognize a variety of patterns and trends including repeating and growing patterns. <br> (A2) Explore extending and comparing arithmetic and geometric sequences. <br> (A3) Develop and test generalizations of patterns and relationships. |
| 1.2 Represent and analyze quantitative relationships in a variety of ways. | a. Demonstrate the equivalence of both sides of an equation. | (A4) Use equations to describe the rules for number patterns and to model word problems. <br> (A5) Demonstrate equivalence with the commutative, associative and distributive properties of whole numbers. |
| 1.3 Use operations, properties and algebraic symbols to determine equivalence and solve problems. | a. Represent possible values using symbols. | (A6) Use variables to represent quantities in expressions and number sentences. |


| Grade 4 <br> NUMERICAL <br> can be expressed numerically in multiple ways in order to make connections and <br> simplify calculations using a variety of strategies, tools and technologies. |  |  |
| :--- | :--- | :--- |
| How are quantitative relationships represented by numbers? |  |  |


|  |  | (N12) Determine and discuss the reasonableness of an answer and explain why a particular estimation strategy will result in an over- or underestimate. <br> (N13) Write and solve multistep word problems, including problems with extraneous information. |
| :---: | :---: | :---: |
|  | b. Use number patterns, basic facts, rectangular arrays, place value models and the distributive property to multiply and divide. | (N14) Develop fluency with multiplication and division fact families for all factors 1 through 10. <br> (N15) Relate multiplication and division to models with groups and rectangular arrays and begin to identify prime and composite numbers. <br> (N16) Explore the property of zero in multiplication and its implication in division. <br> (N17) Identify the appropriate operation and write a word problem to match a given addition, subtraction, multiplication or division number sentence and write the matching number sentence to solve a word problem. <br> (N18) Use arrays and explore using the distributive property $10 \times(5+4)=(10 \times 5)+$ ( $10 \times 4$ ) to estimate, multiply and divide twoand three-digit numbers by one-digit factors. |
|  | c. Add and subtract fractions and mixed numbers with like and unlike denominators using models, pictures and number sentences. | (N19) Solve problems involving the addition and subtraction of fractions with like denominators. <br> (N20) Use models and pictures to estimate a reasonable answer when adding or subtracting decimals, fractions and mixed numbers. <br> (N21) Write and solve word problems involving decimals, fractions and mixed numbers, identify reasonable answers, and match equations to the problems. |


| Grade 4 <br> GEOMETRY AND MEASUREMENT <br> Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How do geometric relationships and measurements help us to solve problems and make sense of our world? |  |  |
| Students Should... | Performance Standards | Expected Performances |
| 3.1 Use properties and characteristics of two- and threedimensional shapes and geometric theorems to describe relationships, communicate ideas and solve problems. | a. Describe geometric properties of polygons and solids. | (G1) Build, draw, describe and classify twoand three-dimensional figures. <br> (G2) Analyze two-dimensional shapes and determine lines of symmetry and congruence. (G3) Identify translations, reflections and rotations in geometric designs. |
| 3.2 Use spatial reasoning, location and geometric relationships to solve problems. | a. Find possible pathways between two points using maps that are based on the rectangular coordinate system. | (G4) Create and read maps and use coordinate systems to specify locations. |
| 3.3 Develop and apply units, systems, formulas and appropriate tools to estimate and measure. | a. Recognize that patterns exist between measurements of length, perimeter and area of squares and rectangles. | (G5) Explore converting inches to feet and feet to yards. <br> (G6) Solve practical problems that involve estimation and measurement of length, perimeter, area, volume, capacity, weight and temperature. <br> (G7) Explore relationships between the lengths of sides of rectangles and their areas and perimeters and generalize the patterns as simple formulas. |
|  | b. Make precise measurements and use benchmarks to estimate measures. | (G8) Identify and use the appropriate customary and metric units and tools for measuring length, perimeter, area, weight, time, temperature, volume and capacity. <br> (G9) Explore converting from one unit to another when measuring time and solve problems that involve elapsed time using clocks and calendars. <br> (G10) Use estimation to predict reasonable answers to measurement problems. <br> (G11) Estimate, draw and measure length to the nearest inch, half-inch and centimeter. |

## Grade 4

WORKING WITH DATA: PROBABILITY AND STATISTICS
Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies.
How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions?

| Students <br> should... | Performance <br> Standards | Expected Performances |
| :--- | :--- | :--- |
| 4.1 Collect, <br> organize and <br> display data <br> using <br> appropriate <br> statistical and <br> graphical <br> methods. | a. Organize and analyze <br> categorical and <br> numerical data. | (D1) Explore a variety of ways to collect, <br> organize, record, analyze and interpret data <br> and identify patterns and trends. <br> (D2) Construct and interpret broken line <br> graphs, line plots, bar graphs, picture <br> graphs, glyphs and simple circle graphs. <br> (D3) Make predictions and defend <br> conclusions based on data. |
| 4.2 Analyze <br> data sets to <br> form <br> hypotheses <br> and make <br> predictions. | a. Describe what is <br> charage" about the <br> set. | (D4) Use the range, mode, median and <br> mean to describe features of a data set. |
| 4.3 Under- <br> stand and a data <br> apply basic <br> concepts of <br> probability. | a. Determine fair <br> situations and good <br> choices based upon the <br> likelihood of an <br> occurrence. | (D5) Identify possible outcomes of events <br> using combinations (where order does not <br> matter) and explore situations resulting in |
| permutations (where order does matter). |  |  |
| (D6) Conduct probability experiments and |  |  |
| express the probability based on possible |  |  |
| outcomes. |  |  |


| Grade 5 <br> ALGEBRAIC REASONING: PATTERNS AND FUNCTIONS <br> Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How do patterns and functions help us describe data and physical phenomena and solve a variety of problems? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 1.1 Understand and describe patterns and functional relationships. | a. Identify trends and make predictions based upon patterns and data displayed in different formats. | (A1) Extend and compare arithmetic and geometric sequences. <br> (A2) Represent geometric and numeric patterns using words, tables, graphs and equations. <br> (A3) Analyze patterns and data to make generalizations and predictions. |
| 1.2 Represent and analyze quantitative relationships in a variety of ways. | a. Recognize that a change in one variable may relate to a change in another variable. | (A4) Describe how a change in one variable relates to a change in a second variable in context. |
| 1.3 Use operations, properties and algebraic symbols to determine equivalence and solve problems. | a. Describe the general relationship between two sets of data using an equation or inequality. | (A5) Represent mathematical relationships using variables in expressions, equations and inequalities. <br> (A6) Model and solve one-step equations using materials that model equivalence. |


| Grade 5 <br> NUMERICAL AND PROPORTIONAL REASONING: Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
|  |  |  |
| How are quantitative relationships represented by numbers? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 2.1 Understand that a variety of numerical representations can be used to describe quantitative relationships. | a. Extend whole number place value patterns, models and notations to include decimals, which are fractions that have denominators that 10. | (N1) Identify, round, order and compare whole numbers to $1,000,000$ using place value models, diagrams and number lines. <br> (N2) Express numbers in expanded and regrouped forms and use the numbers to support computational strategies. <br> (N3) Solve problems involving finding 10, 100 and 1000 more and less than a number. <br> (N4) Estimate products and missing factors using multiples of 10, 100 and 1000. <br> (N5) Use models to extend whole number place value concepts and patterns to decimals. <br> (N6) Explore numbers less than zero and extend the number line to introduce the concept of integers within practical applications. |
|  | b. Classify numbers by their factors. | (N7) Use rectangular arrays to identify factor pairs and to classify numbers as prime, composite and perfect squares. <br> (N8) Explore divisibility rules and patterns with remainders. |
|  | c. Express numbers as equivalent fractions, decimals or percents. | (N9) Represent a rational number in its equivalent fraction, decimal, ratio and percent forms with models, number patterns and common factors. <br> (N10) Construct and use models and pictures to add and subtract fractions, decimals and mixed numbers with like and unlike denominators. <br> (N11) Use equivalence and substitution with common denominators when adding and subtracting. <br> (N12) Construct and use models and pictorial representations to multiply common fractions and mixed numbers. |
|  | d. Represent ratios and proportions and solve problems using models and pictures. | (N13) Build models to identify and compare ratios and describe quantitative relationships using fraction and decimal equivalents. <br> (N14) Write division problems in fraction form and round the fraction form to estimate an answer to a division problem. <br> (N15) Use ratios and proportions to solve practical problems such as interpreting maps and |


|  |  | scale drawings or identifying probability. |
| :--- | :--- | :--- |
| 2.2 Use | a. Estimate and | (N16) Choose and use benchmarks to |
| numbers and | compute using | approximate locations on number lines and |
| their properties | models and | coordinate grids. |
| to compute | pictures. | (N17) Estimate and use counting, grouping of |
| flexibly and |  | objects, number patterns, equivalent ratios and |
| fluently, and to |  | division to find fractional parts of a set of |
| reasonably |  | objects. |
| estimate |  | (N18) Develop strategies, using place value |
| measures and |  | relationships, inverse operations and |
| quantities. |  | commutative, associative and distributive |
|  |  | properties, to simplify computations with two-, |
|  |  | three-, and four-digit numbers and money |
|  |  | amounts. |
|  |  | (N19) Use estimation to predict results and to |
|  |  | recognize when an answer is or is not reasonable. |
|  |  | (N20) Explain when an estimation strategy will |
|  |  | result in an over- or underestimate. |
|  |  | (N21) Create and solve multistep problems and |
|  |  | explore order of operations in the context of |
|  |  | practical situations. |


| Grade 5 <br> GEOMETRY AND MEASUREMENT <br> Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How do geometric relationships and measurements help us to solve problems and make sense of our world? |  |  |
| Students <br> Should... | Performance Standards | Expected Performances |
| 3.1 Use <br> properties and <br> characteristics <br> of two- and <br> three- <br> dimensional <br> shapes and <br> geometric <br> theorems to <br> describe <br> relationships, <br> communicate <br> ideas and solve <br> problems. | a. Use geometric relationships to describe polygons and solids. | (G1) Use geometric relationships such as parallel, perpendicular and congruent to describe the attributes of sets and subsets of shapes and solids. <br> (G2) Make and test conjectures about geometric relationships. |
|  | b. Recognize that changes in the perimeter of a polygon may affect its area, and changes in area may affect the perimeter. | (G3) Explore the relationship between area and perimeter when the dimensions of a polygon change. <br> (G4) Develop formulas to find the perimeter and area of squares, rectangles and triangles. |
| 3.2 Use spatial reasoning, location and geometric relationships to solve problems. | a. Identify, describe and build nets for solid figures and objects. | (G5) Represent the surface of threedimensional objects through the use of twodimensional nets. <br> (G6) Investigate and develop strategies to determine the volume of rectangular solids. |
|  | b. Determine geometric relationships through spatial visualization. | (G7) Plot points on the rectangular coordinate system and estimate and determine the distance between points. |
| 3.3 Develop and apply units, systems, formulas and appropriate tools to estimate and measure. | a. Solve problems in the measure of time and in the conversion of units of length in the customary and metric systems using specific ratios. | (G8) Solve length problems involving conversion of measure within the customary and metric systems. <br> (G9) Solve problems involving the conversion of measure of time and elapsed time (days, hours, minutes and seconds). (G10) Estimate and choose appropriate units and tools to measure and solve a variety of problems involving length, perimeter, area, volume, capacity, mass, time, angle and temperature. |


| Grade 5 <br> WORKING WITH DATA: PROBABILITY AND STATISTICS <br> Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 4.1 Collect, organize and display data using appropriate statistical and graphical methods. | a. Differentiate between numerical and categorical data and their appropriate representations. | (D1) Construct and interpret broken line graphs, line plots, bar graphs, picture graphs, simple circle graphs, and stem and leaf plots and evaluate how well each kind of display represents the features of the data. |
| 4.2 Analyze <br> data sets to <br> form <br> hypotheses and make predictions. | a. Examine different data collection methods and their effects. | (D2) Design and conduct surveys and samplings to collect data that represent a general population. <br> (D3) Explore how a change in an outlier can change the measures of central tendency. |
| 4.3 Understand and apply basic concepts of probability. | a. Relate the likelihood of an event to a numerical value. | (D4) Identify possible outcomes and express the likelihood of events as a fraction. (D5) Design and conduct probability experiments and games of chance. (D6) Make and test predictions of probability and fairness. |


| Grade 6 <br> ALGEBRAIC REASONING: PATTERNS AND FUNCTIONS <br> Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How do patterns and functions help us describe data and physical phenomena and solve a variety of problems? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 1.1 Understand and describe patterns and functional relationships. | a. Identify relationships and make generalizations through the use of patterns. | (A1) Describe, analyze and extend numeric, geometric and statistical patterns and use them to identify trends and justify predictions. |
| 1.2 Represent and analyze quantitative relationships in a variety of ways. | a. Represent and analyze mathematical relationships with the help of tables, graphs, equations and inequalities. | (A2) Determine the nature of changes in linear relationships using graphs, tables and equations. <br> (A3) Represent numerical and contextual situations with algebraic expressions, equations and inequalities. |
| 1.3 Use operations, properties and algebraic symbols to | a. Solve real-world problems using algebraic methods. | (A4) Use variables as placeholders, to denote a pattern, to write a formula and to represent a function or relation. <br> (A5) Evaluate algebraic expressions and formulas using substitution. |
| equivalence and solve problems. | b. Demonstrate how to maintain equivalence in equations. | (A6) Model and solve one-step linear equations by maintaining equivalence. |


| Grade 6 <br> NUMERICAL AND PROPORTIONAL REASONING: Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How are quantitative relationships represented by numbers? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 2.1 Understand that a variety of numerical representations can be used to describe quantitative relationships. | a. Relate whole numbers, fractions, decimals and integers to number lines, scales, the coordinate plane and problem-solving situations. | (N1) Locate, order and compare whole numbers, fractions, decimals and integers on number lines, scales and the coordinate grid. <br> (N2) Explain orally and in writing when a situation requires an exact answer or when an estimate is sufficient. |
|  | b. Express place value patterns using exponents to write powers of ten. | (N3) Recognize place value patterns when multiplying and dividing decimals by powers of 10 . <br> (N4) Compare large numbers using expanded forms and powers of ten. <br> (N5) Develop, describe and use a variety of ways to estimate and calculate with large numbers and connect the strategies to powers of ten. |
|  | c. Interpret and connect fraction notation to division. | (N6) Use models and common factors to identify equivalent fractions and their decimal representations. <br> (N7) Determine the decimal equivalents of fractions. <br> (N8) Recognize that multiplication by a unit fraction is equivalent to dividing by the fraction's denominator. |
|  | d. Compare quantities and solve problems using ratios, rates and percents. | (N9) Estimate and find percents using benchmarks and number patterns. <br> (N10) Convert between rates using ratios and proportions. <br> (N11) Solve problems involving ratios, proportions and percents. |

$\left.\begin{array}{|l|l|l|}\hline \begin{array}{l}\text { 2.2 Use numbers and } \\ \text { their properties to } \\ \text { compute flexibly and } \\ \text { fluently, and to } \\ \text { reasonably estimate } \\ \text { measures and quantities. }\end{array} & \begin{array}{l}\text { a. Solve problems using } \\ \text { a variety of } \\ \text { computational strategies, } \\ \text { including the use of } \\ \text { calculators. }\end{array} & \begin{array}{l}\text { (N12) Estimate and predict } \\ \text { reasonable answers and recognize } \\ \text { and explain when an estimate will } \\ \text { be more or less than an exact } \\ \text { answer. } \\ \text { (N13) Use a variety of } \\ \text { computational strategies (mental } \\ \text { computation, paper-and-pencil and } \\ \text { calculator) to add, subtract, } \\ \text { multiply and divide multidigit }\end{array} \\ & & \begin{array}{l}\text { numbers in the context of } \\ \text { multistep word and practical } \\ \text { problems. } \\ \text { (N14) Apply the order of }\end{array} \\ & & \begin{array}{l}\text { operations and algebraic properties } \\ \text { (associative, commutative, } \\ \text { distributive, inverse operations and } \\ \text { additive and multiplicative } \\ \text { identities) to estimate and solve }\end{array} \\ \text { multistep problems. } \\ \text { (N15) Use factors of composite }\end{array}\right\}$

| Grade 6 <br> GEOMETRY AND MEASUREMENT <br> Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How do geometric relationships and measurements help us to solve problems and make sense of our world? |  |  |
| Students <br> Should... | Performance Standards | Expected Performances |
| 3.1 Use <br> properties and characteristics of two- and threedimensional shapes and geometric theorems to describe relationships, communicate ideas and solve problems. | a. Classify polygons according to their properties. | (G1) Use the relationships of sides and angles to classify sets and subsets of polygons. <br> (G2) Make and test conjectures about side and angle relationships and congruence. |
|  | b. Examine the relationships between the measures of area of twodimensional objects and volume of threedimensional objects. | (G3) Use the rectangle as a basic shape to model and develop formulas for the area of triangles, parallelograms, trapezoids and circles. <br> (G4) Recognize the relationships among radius, diameter, circumference and area of circles. <br> (G5) Develop and use strategies to determine the volume of rectangular solids and cylinders. |
| 3.2 Use spatial reasoning, location and geometric relationships to solve problems. | a. Construct similar polygons on coordinate grids. | (G6) Explore similarity of polygons as a result of dilations (a reduction or enlargement) and their effects on their measurements. |
| 3.3 Develop and apply units, systems, formulas and appropriate tools to estimate and measure. | a. Solve problems involving measurement through the use of a variety of tools, techniques and strategies. | (G7) Estimate and determine length, area, volume, mass and angle measures. <br> (G8) Select and use appropriate units, strategies and tools to measure and solve problems involving length, perimeter, area, volume, capacity, weight, mass, temperature and angles. |
|  | b. Use specific ratios to convert between measure of length, area, volume, mass and capacity in the customary and metric systems. | (G9) Use different ratios to convert between units of length, area and volume in the customary and metric systems. (G10) Recognize and use powers of ten as conversion ratios in the metric system. |


| Grade 6 <br> WORKING WITH DATA: PROBABILITY AND STATISTICS <br> Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions? |  |  |
| Students should... | Content Standards | Expected Performances |
| 4.1 Collect, organize and display data using appropriate statistical and graphical methods. | a. Display and compare sets of data using various systematic or graphical representations. | (D1) Compare sets of data graphically using histograms, double bar graphs, back-to-back stem and leaf plots and scatter plots. <br> (D2) Construct circle graphs and recognize that they represent data proportionally. <br> (D3) Use systematic listing and counting strategies to solve problems. |
| 4.2 Analyze data sets to form hypotheses and make predictions. | a. Describe the shape of data sets using the measures of spread and central tendency. | (D4) Describe the shape of data sets using measures of spread (range and outliers) and central tendency (mode, median and mean). <br> (D5) Recognize that changes in a data set can affect the mode, median, mean and range. |
| 4.3 Understand and apply basic concepts of probability. | a. Understand that probabilities are more reliable to use as predictors when there is a large number of trials. | (D6) Explore the relationship between the number of trials in an experiment and the predicted outcomes. <br> (D7) Design and conduct probability experiments and make predictions about outcomes that are equally likely or not equally likely. |
|  | b. Express probability using various numerical representations. | (D8) Express probabilities as fractions, ratios, decimals and percents. |


| ALGEBRAIC REASONING: <br> Patterns and functional relationships can be represented and analyzed using <br> a variety of strategies, tools and technologies. |  |
| :--- | :--- | :--- |
| How do patterns and functions help us describe data and physical |  |
| phenomena and solve a variety of problems? |  |


| NUMERICAL AND PROPORTIONAL REASONING: Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How are quantitative relationships represented by numbers? |  |  |
| Students Should... | Performance Standards | Expected Performances |
| 2.1 Understand that a variety of numerical representations can be used to describe quantitative relationships. | a. Represent realworld situations and solutions to problems using the appropriate symbolic form (fractions, decimals or percents). | (N1) Rewrite a rational number in its equivalent fraction, decimal, ratio and percent forms with number patterns and common factors. <br> (N2) Identify and classify fractions as terminating or repeating decimals. <br> (N3) Estimate and perform computations with fractions, decimals, mixed numbers, improper fractions, ratios, proportions and percents. <br> (N4) Multiply and divide mixed numbers and decimals using the distributive property. <br> (N5) Use and describe appropriate methods to divide by a fraction or a decimal. <br> (N6) Solve practical problems involving rates, scale factors, mixtures and percents with proportions. <br> (N7) Estimate to predict outcomes and determine reasonableness of results, and describe whether an estimate is an over- or underestimate. |
|  | b. Understand the use of scientific notation as related to powers of ten as an efficient method for writing and comparing very large numbers. | (N8) Use powers of ten and positive exponents to express and compare magnitude of very large numbers and connect to scientific notation. <br> (N9) Develop, describe and use a variety of methods to estimate and calculate with very large numbers. |
|  | c. Use percents to make comparisons between groups of unequal size. | (N10) Estimate and find percents, including percents greater than 100 percent and less than 1 percent using number patterns and the distributive property. <br> (N11) Find what percent one amount is of another amount using a variety of strategies. |
| 2.2 Use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities. | a. Extend the operations of addition, subtraction, multiplication and division to negative numbers. | (N12) Solve problems with positive and negative numbers using models and number lines. <br> (N13) Use the order of operations to compute and solve a variety of multistep problems, including those with parentheses and exponents. <br> (N14) Explore absolute value while solving problems involving distance. |


| Grade 7 <br> GEOMETRY AND MEASUREMENT <br> Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How do geometric relationships and measurements help us to solve problems and make sense of our world? |  |  |
| Students Should... | Performance Standards | Expected Performances |
| 3.1 Use <br> properties and characteristics of two- and threedimensional shapes and geometric theorems to describe relationships, communicate ideas and solve problems. | a. Describe and classify polygons according to their transformational properties. | (G1) Identify which classes of polygons have line and/or rotational symmetry. <br> (G2) Use rectangular grids to represent polygons and perform transformations (translations, rotations, reflections and dilations) on these polygons. <br> (G3) Describe the effect of transformations on polygons with line and/or rotational symmetry. |
| 3.2 Use spatial reasoning, location and geometric relationships to solve problems. | a. Understand how threedimensional objects can be represented in two dimensions using base plans (footprints), orthogonal views, nets and isometric drawings. | (G4) Draw and interpret nets, cross-sections and front, side and top views of various solids. <br> (G5) Develop and use strategies to determine the surface area of threedimensional objects. |
| 3.3 Develop and apply units, systems, formulas and appropriate tools to estimate and measure. | a. Solve geometric and measurement problems through the use of a variety of tools, techniques and strategies. | (G6) Use estimation and measurement strategies to solve problems involving the areas of irregular polygons and volumes of irregular solids. |

## Grade 7

WORKING WITH DATA: PROBABILITY AND STATISTICS
Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies.
How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions?

| Students <br> should... | Content Standards | Expected Performances |
| :--- | :--- | :--- |
| 4.1 Collect, <br> organize and <br> display data <br> using <br> appropriate <br> statistical and <br> graphical <br> methods. | a. Select the appropriate <br> visual representation of <br> data based on the kind of <br> data collected and the <br> purpose for its use. | (D1) Formulate questions, design surveys <br> and samplings, organize and analyze <br> gathered data and defend the analysis. <br> (D2) Organize and display data using <br> appropriate graphical representations and <br> make and defend predictions based on <br> patterns and trends. |
| 4.2 Analyze <br> data sets to <br> form <br> hypotheses <br> and make <br> predictions. | a. Understand that <br> measures of central <br> tendency and spread can <br> be used to describe data <br> sets and justify <br> conclusions. | (D3). Find, use and interpret measures of <br> central tendency and spread, including <br> mode, median, mean, range and outliers. <br> (D4) Compare two sets of data based on <br> their distributions and measures of central <br> tendency. |
| 4.3 Under- <br> stand and <br> apply basic <br> concepts of <br> probability. | a. Compare and <br> determine experimental <br> and theoretical <br> probabilities. | (D5) Identify the two ways of obtaining <br> probabilities: by gathering data from <br> experiments (experimental probability); and <br> by analyzing the possible and likely |


| Grade 8 <br> ALGEBRAIC REASONING: PATTERNS AND FUNCTIONS <br> Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How do patterns and functions help us describe data and physical phenomena and solve a variety of problems? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 1.1 Understand and describe patterns and functional relationships. | a. Analyze physical phenomena, functions and patterns to identify relationships and make generalizations. | (A1) Write recursive and explicit functions to generalize patterns. <br> (A2) Identify relationships that are linear and nonlinear and compare and contrast their properties using tables, graphs, equations and verbal descriptions. <br> (A3) Recognize and solve problems of direct variation. |
| 1.2 Represent and analyze quantitative relationships in a variety of ways. | a. Describe the effects of characteristics of linear relationships on the way the relationships are represented verbally and in tables, graphs and equations. | (A4) Determine the constant rate of change in a linear relationship and recognize this as the slope of a line. <br> (A5) Compare and contrast the graphs of lines with the same slope versus those with different slopes. <br> (A6) Interpret slope and y-intercepts from contextual situations, graphs and linear equations. <br> (A7) Given two linear relationships in context, recognize that they may have a common solution. |
| 1.3 Use operations, properties and algebraic symbols to determine equivalence and solve problems. | a. Solve problems using various algebraic methods and properties. | (A8) Solve multistep equations using algebraic properties. <br> (A9) Use tables, graphs and equations to represent mathematical relationships and solve real-world problems. |


| Grade 8 <br> NUMERICAL AND PROPORTIONAL REASONING: Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How are quantitative relationships represented by numbers? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 2.1 Understand that a variety of numerical representations can be used to describe quantitative relationships. | a. Compare and order integers, powers and roots using number lines and grids. | (N1) Compare, locate, label and order rational numbers on number lines, scales, coordinate grids and measurement tools. <br> (N2) Identify another rational number between any two rational numbers. <br> (N3) Solve a variety of problems involving integers, powers, roots and scientific notation. |
|  | b. Extend the understanding of scientific notation to very small numbers. | (N4) Use powers of ten and negative exponents to write decimal fractions. <br> (N5) Use powers of ten and positive and negative exponents to express and compare magnitude of very large and very small numbers and connect to scientific notation. (N6) Find the results of multiplication and division with powers of ten using patterns in operating with exponents. <br> (N7) Develop, describe and use a variety of methods to operate with very large and very small numbers. |
| 2.2 Use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities. | a. Solve problems involving fractions, decimals, ratios and percents. | (N8) Estimate and solve problems involving percent of increase and decrease. |
|  | b. Make generalizations about operations with very large and very small numbers. | (N9) Use the rules for exponents to multiply and divide with powers of ten, including negative exponents. <br> (N10) Develop, describe and use a variety of methods to estimate and calculate mentally with very large and very small numbers. |
|  | c. Connect the exponential growth and decay models to repeated multiplication by the same factor. | (N11) Solve problems that involve repetitive patterns and iterations, such as compound interest, using tables, spreadsheets and calculators. |


| Grade 8 <br> GEOMETRY AND MEASUREMENT <br> Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How do geometric relationships and measurements help us to solve problems and make sense of our world? |  |  |
| Students <br> Should... | Performance Standards | Expected Performances |
| 3.1 Use <br> properties and characteristics of two- and threedimensional shapes and geometric theorems to describe relationships, communicate ideas and solve problems. | a. Explore the relationships among sides, angles, perimeters, areas, surface areas and volumes of congruent and similar polygons and solids. | (G1) Explore the effect of scale factors on the length, area and volume ratios of similar polygons, circles and solids. <br> (G2) Make and test conjectures about the relationships among angles, sides, perimeters and areas of congruent and similar polygons, including the Pythagorean Theorem. |
| 3.2 Use spatial reasoning, location and geometric relationships to solve problems. | a. Model geometric relationships in a variety of ways. | (G3) Use coordinate geometry to explore and test geometric relationships of parallel and perpendicular lines and polygons and their transformations. |
| 3.3 Develop and apply units, systems, formulas and appropriate tools to estimate | a. Use a variety of concrete methods, including displacement, to find volumes of solids. | (G4) Develop measurement strategies to find the surface area and volume of pyramids, cones, spheres and irregular solids. (G5) Use estimation and measurement strategies to solve problems involving the volumes of solids. |
|  | b. Solve problems involving measurement through the use of appropriate tools, techniques and strategies. | (G6) Use the Pythagorean Theorem to solve indirect measurement problems. <br> (G7) Describe the accuracy of estimates and measures and the precision of measurement tools. <br> (G8) Solve dimensional analysis problems. |


| Grade 8 <br> WORKING WITH DATA: PROBABILITY AND STATISTICS <br> Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions? |  |  |
| Students should... | Content Standards | Expected Performances |
| 4.1 Collect, organize and display data using appropriate statistical and graphical methods. | a. Construct appropriate representations of data based on the size and kind of data set and the purpose for its use. | (D1) Collect, organize, display, compare and analyze large data sets. <br> (D2) Construct a variety of data displays, including box-and-whisker plots, and identify where measures of central tendency and dispersion are found in graphical displays. |
| 4.2 Analyze data sets to form hypotheses and make predictions. | a. Make and evaluate statistical claims and justify conclusions with evidence. | (D3) Make predictions from scatter plots using or estimating a line-of-best-fit. <br> (D4) Make inferences and evaluate reasonable hypotheses based on experimental data. <br> (D5) Analyze and interpret data using descriptive statistics, including range, mode, median, quartiles, outliers and mean. (D6) Determine the accuracy of statistical claims. <br> (D7) Describe the role of random sampling, random number generation and the effects of sample size in statistical claims. |
| 4.3 Understand and apply basic concepts of probability. | a. Determine possible outcomes using a variety of counting techniques. | (D8) Distinguish between combinations and permutations as ways to predict possible outcomes in certain situations. (D9) Use combinations and permutations, trees and networks (counting strategies) in a variety of contexts, and identify when order is irrelevant in determining a solution. |


| 9-12 Core <br> ALGEBRAIC REASONING: PATTERNS AND FUNCTIONS <br> Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How do patterns and functions help us describe data and physical phenomena and solve a variety of problems? |  |  |
| Students should... | Performance Standards | Expected Performances |
| 1.1 Understand and describe patterns and functional relationships. | a. Describe relationships and make generalizations about patterns and functions. | (A1) Identify, describe, create and generalize numeric, geometric and statistical patterns with tables, graphs, words and symbolic rules. <br> (A2) Make and justify predictions based on patterns. <br> (A3) Identify the characteristics of functions and relations, including domain and range. <br> (A4) Describe and compare properties and classes of linear, quadratic and exponential functions. |
| 1.2 Represent and analyze quantitative relationships in a variety of ways. | a. Represent and analyze linear and nonlinear functions and relations symbolically and with tables and graphs. | (A5) Represent functions and relations on the coordinate plane. <br> (A6) Identify an appropriate symbolic representation for a function or relation displayed graphically or verbally. <br> (A7) Recognize and explain the meaning of the slope and $x$ - and $y$-intercepts as they relate to a context, graph, table or equation. <br> (A8) Evaluate and interpret the graphs of linear, exponential and polynomial functions. |
| 1.3 Use operations, properties and algebraic symbols to determine equivalence and solve problems. | a. Manipulate equations, inequalities and functions to solve problems. | (A8) Model and solve problems with linear, quadratic and absolute value equations and linear inequalities. <br> (A9) Determine equivalent representations of an algebraic equation or inequality to simplify and solve problems. <br> (A10) Solve systems of two linear equations using algebraic or graphical methods. |


| Grade 9-12 Core <br> NUMERICAL AND PROPORTIONAL REASONING: Quantitative relationships <br> can be expressed numerically in multiple ways in order to make connections and <br> simplify calculations using a variety of strategies, tools and technologies. |  |
| :--- | :--- | :--- |
| How are quantitative relationships represented by numbers? |  |


| Grade 9-12 CORE <br> GEOMETRY AND MEASUREMENT <br> Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How do geometric relationships and measurements help us to solve problems and make sense of our world? |  |  |
| Students Should... | Performance Standards | Expected Performances |
| 3.1 Use properties and characteristics of two- and threedimensional shapes and geometric theorems to describe relationships, | a. Investigate relationships among plane and solid geometric figures using geometric models, constructions and tools. | (G1) Use models and constructions to make, test and summarize conjectures involving properties of geometric figures. (G2) Use geometric properties to solve problems in two and three dimensions. <br> (G3) Determine and compare properties of classes of polygons. |
|  | b. Develop and evaluate mathematical arguments using reasoning and proof. | (G4) Recognize the validity of an argument. <br> (G5) Create logical arguments to solve problems and determine geometric relationships. |
| 3.2 Use spatial reasoning, location and geometric relationships to solve problems. | a. Verify geometric relationships using algebra, coordinate geometry and transformations. | (G6) Interpret geometric relationships using algebraic equations and inequalities, and vice versa. <br> (G7) Describe how a change in measurement of one or more parts of a polygon or solid may affect its perimeter, area, surface area and volume and make generalizations for similar figures. <br> (G8) Apply transformations to plane figures to determine congruence, similarity, symmetry and tessellations. |
| 3.3 Develop and apply units, systems, formulas and appropriate tools to estimate and measure. | a. Solve a variety of problems involving one-, two- and threedimensional measurements using geometric relationships and trigonometric ratios. | (G9) Select appropriate units, scales, degree of precision, and strategies to determine length, angle measure, perimeter, circumference and area of plane geometric figures. <br> (G10) Use indirect methods including the Pythagorean Theorem, trigonometric ratios and proportions in similar figures to solve a variety of measurement problems. <br> (G11) Judge the reasonableness of answers to direct and indirect measurement problems. <br> (G12) Use two-dimensional representations and formal and informal methods to solve surface-area and volume problems. |


| Grade 9-12 Core <br> WORKING WITH DATA: PROBABILITY AND STATISTICS <br> Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions? |  |  |
| Students should... | Content Standards | Expected Performances |
| 4.1 Collect, organize and display data using appropriate statistical and graphical methods. | a. Create the appropriate visual or graphical representation of real data. | (D1) Collect real data and create meaningful graphical representations of the data. <br> (D2) Develop, use and explain applications and limitations of linear and nonlinear models and regression in a variety of contexts. |
| 4.2 Analyze <br> data sets to form hypotheses and make predictions. | a. Analyze real-world problems using statistical techniques. | (D3) Estimate an unknown value between data points on a graph (interpolation) and make predictions by extending the graph (extrapolation). <br> (D4) Use data from samples to make inferences about a population and determine whether claims are reasonable or false. <br> (D5) Determine and use measures of spread and central tendency to describe and compare sets of data. |
| 4.3 Understand and apply basic concepts of probability. | a. Understand and apply the principles of probability in a variety of situations. | (D6) Determine outcomes and solve problems involving the probabilities of events. <br> (D7) Explore the concepts of conditional probability in real-world contexts. <br> (D8) Apply theoretical and experimental probabilities appropriately to solve problems and predict experimental results. |

## 9-12 Extended

ALGEBRAIC REASONING: PATTERNS AND FUNCTIONS
Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.

| How do patterns and functions help us describe data and physical phenomena and solve a variety of problems? |  |  |
| :---: | :---: | :---: |
| Students should... | Performance Standards | Expected Performances |
| 1.1 Understand and describe patterns and functional relationships. | a. Model real-world situations and make generalizations about mathematical relationships using a variety of patterns and functions. | (AE 1) Describe and compare properties and classes of functions, including exponential, polynomial, rational, logarithmic and trigonometric. <br> (AE 2) Analyze essential relations in a problem to determine possible functions that could model the situation. <br> (AE 3) Explore conic sections and their applications graphically and symbolically. (AE 4) Solve problems involving financial applications including compound interest, amortization of loans, and investments. <br> (AE 5) Solve problems involving direct and inverse variation. <br> (AE 6) Understand and use optimization strategies, including linear programming. <br> (AE 7) Apply the concepts of limits to sequences and asymptotic behavior of functions. |
| 1.2 Represent and analyze quantitative relationships in a variety of ways. | a. Relate the behavior of functions and relations to specific parameters and determine functions to model real-world situations. | (AE 8) Relate the graphical representation of a function to its function family and find equations, intercepts, maximum or minimum values, asymptotes and line of symmetry for that function. <br> (AE 9) Recognize the effect of changes in parameters on the graphs of functions or relations. <br> (AE 10) Recognize that the slope of the tangent line to a curve represents the rate of change. <br> (AE 11) Represent functions and relations with polar coordinates and in the complex plane. |
| 1.3 Use operations, properties and algebraic symbols to determine equivalence and solve problems. | a. Use and extend algebraic concepts to include real and complex numbers, vectors and matrices. | (AE 12) Determine equivalent representations of an algebraic equation or inequality to simplify and solve problems. <br> (AE 13) Combine, compose and invert functions. <br> (AE 14) Use logarithms, vectors and matrices to solve problems. |


| Grade 9-12 Extended <br> NUMERICAL AND PROPORTIONAL REASONING: Quantitative relationships <br> can be expressed numerically in multiple ways in order to make connections and <br> simplify calculations using a variety of strategies, tools and technologies. |  |  |
| :--- | :--- | :--- |
| How are quantitative relationships represented by numbers? |  |  |
| Students <br> should... |  | Performance |
| Standards |  |  |$\quad$| Expected Performances |
| :--- |
| 2.1 Understand <br> that a variety <br> of numerical <br> representations <br> can be used to <br> describe |
| quantitative <br> relationships. |
| understanding of <br> number to include the <br> set of complex <br> numbers. |
|  |


| Grade 9-12 EXTENDED <br> GEOMETRY AND MEASUREMENT <br> Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How do geometric relationships and measurements help us to solve problems and make sense of our world? |  |  |
| Students Should... | Performance Standards | Expected Performances |
| 3.1 Use properties and characteristics of two- and threedimensional shapes and geometric theorems to describe | a. Use methods of deductive and inductive reasoning to make, test and validate geometric conjectures. | (GE 1) Recognize the relationships between a conditional statement and its converse, inverse and contrapositive. <br> (GE 2) Test the validity of logical arguments. <br> (GE 3) Use deductive arguments, including direct and indirect proofs, to develop an understanding of an axiomatic approach to geometry. |
| relationships, communicate ideas and solve problems. | b. Explore nonEuclidean geometries. | (GE 4) Recognize that the familiar geometry of Euclid is based on a particular set of axioms and that a different set of axioms would lead to a different geometry. |
| 3.2 Use spatial reasoning, location and geometric relationships to solve problems. | a. Use a variety of coordinate systems and transformations to solve geometric problems in two- and threedimensions using appropriate tools and technologies | (GE 5) Visualize three-dimensional objects from different perspectives and analyze cross-sections, surface area and volume. (GE 6) Use Cartesian, navigational, polar and spherical systems to represent, analyze and solve geometric and measurement problems. <br> (GE 7) Represent translations, reflections, rotations and dilations of plane figures using sketches, coordinates, vectors, function notation and matrices to examine the effects of transformations and their composites and to solve related geometric problems. |
| 3.3 Develop and apply units, systems, formulas and appropriate tools to estimate and measure. | a. Approximate measurements that cannot be directly determined with some degree of precision using appropriate tools, techniques and strategies. | (GE 8) Use successive approximation, upper and lower bounds, and limits to solve measurement problems. <br> (GE 9) Use properties of similarity and techniques of trigonometry to make indirect measurements of lengths and angles to solve a variety of problems. |


| Grade 9-12 Extended <br> WORKING WITH DATA: PROBABILITY AND STATISTICS <br> Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies. |  |  |
| :---: | :---: | :---: |
| How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions? |  |  |
| Students should... | Content Standards | Expected Performances |
| 4.1 Collect, organize and display data using appropriate statistical and graphical methods. | a. Model real data graphically using appropriate tools, technology and strategies. | (DE 1) Investigate and solve relevant problems by designing statistical experiments and collecting, organizing, displaying and analyzing data in tabular, graphical and symbolic forms. <br> (DE 2) Apply and defend regression models for bivariate data and use them to formulate predictions. <br> (DE 3) Recognize the limitations of mathematical models based on sample data as representations of real-world situations. |
| 4.2 Analyze data sets to form hypotheses and make predictions. | a. Describe and analyze sets of data using statistical models. | (DE 4) Determine statistical measures to describe univariate data. <br> (DE 5) Describe characteristics of sampling methods and analyze the effects of random versus biased sampling. |
| 4.3 Understand and apply basic concepts of probability. | a. Solve problems using the methods of discrete mathematics. | (DE 6) Understand and use permutations, combinations, recursion and mathematical induction to solve problems. <br> (DE 7) Solve problems using finite graphs. |
|  | b. Make statistical inferences through the use of probability. | (DE 8) Explore the characteristics and applications of the normal distribution and standardized scores. <br> (DE 9) Construct and interpret confidence intervals. <br> (DE 10) Explore a variety of statistical tests such as chi-squares and t-tests and understand the meaning of hypothesis testing. <br> (DE 11) Use relative frequency and expected values to represent and solve problems involving uncertainty. |

